PERGAMON

International Journal of Solids and Structures 36 (1999) 2757–2771

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

Woven fabric composite material model with material nonlinearity for nonlinear finite element simulation

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Received 20 September 1997; in revised form 22 March 1998

Abstract

The objective of the current investigation is to develop a simple, yet generalized, model which considers the two-dimensional extent of woven fabric, and to have an interface with nonlinear finite element codes. A micromechanical composite material model for woven fabric with nonlinear stress-strain relations is developed and implemented in ABAQUS for nonlinear finite element structural analysis. Within the model a representative volume cell is assumed. Using the iso-stress and iso-strain assumptions the constitutive equations are averaged along the thickness direction. The cell is then divided into many subcells and an averaging is performed again by assuming uniform stress distribution in each subcell to obtain the effective stress–strain relations of the subcell. The stresses and strains within the subcells are combined to yield the effective stresses and strains in the representative cell. Then this information is passed to the finite element code at each material point of the shell element. In this manner structural analysis of woven composites can be performed. Also, at each load increment global stresses and strains are communicated to the representative cell and subsequently distributed to each subcell. Once stresses and strains are associated to a subcell they can be distributed to each constituent of the subcell i.e. fill, warp, and resin. Consequently micro-failure criteria (MFC) can be defined for each constituent of a subcell and the proper stiffness degradation can be modeled if desired. This material model is suitable for implicit and could be modified for explicit finite element codes to deal with problems such as crashworthiness, impact, and failure analysis under static loads.

Nomenclature

a	half length of the representative cell
a_u	half length of undulation in fill direction
a_0	$a-a_u$
b	half width of the representative cell

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0020–7683/99/$ - see front matter
PII: S0020–7683(98)00127–9

b_u half length of undulation in warp direction
b_0 $b - b_u$
E Young's modulus
G shear modulus
H height of the representative cell
H_t height of fiber tows
l_x length of a subcell
l_y width of a subcell
L_x length of the representative cell
L_y width of the representative cell
S_{ij} compliance components in principal material system
V volume of a representative cell
$(\bar{\ })$ average quantities of representative volume cell
$()^t$ quantities of tangential stiffness matrix or compliance matrix
$()^*$ in-plane or out-of-plane strains or stresses in principal material coord. system
$()_i$ in-plane stress or strain components
$()_o$ out-of-plane stress or strain components
$()_f$ quantities of the fill tow
$()_w$ quantities of the warp tow
$()_m$ quantities of the matrix

Greek symbols
ψ_{ij} compliance components of constituents in global coordinate system
Ψ_{ij} compliance components of the whole cell in global coordinate system
θ_f local angle between the fill yarn and global coordinate system
θ_w local angle between the warp yarn and global coordinate system
$()^{(\alpha,\beta)}$ average quantities of a subcell

Subscripts
x, y, z quantities in global coordinate system
$1, 2, 3$ quantities in principal material coordinates

1. Introduction

Analytical models for determination of mechanical properties of woven composites provide a cost-effective tool to determine the effects of several parameters on the mechanical properties. These parameters include fabric weight, constituent volume fraction, yarn undulation, weave style and properties of the constituent materials. The advantage of a micromechanical analysis is even more crucial when dealing with nonlinearity on the constituent level and stiffness degradation due to damage of those constituents (resin, fill, and warp). Three different types of models are available in literature: elementary, laminate theory and numerical models. These models were developed with the aim of determining mechanical properties of woven composites. The numerical models involved the use of the finite element method to analyze the elastic behavior and determine mechanical properties of woven composites. The elementary and laminate theory models, although simple, neglect the two dimensional extent of the fabric.

The finite element method can be used to study the overall behavior of composite structures on the macro level and the material behavior on the constituent level. Whitcomb (1991), Zhang and Harding (1990) and Chapman and Whitcomb (1996) have studied the elastic material properties of woven composites by the finite element method. A more practical issue may arise when studying the global behavior of structures with consideration of the material nonlinearity and progressive damage. Consequently, application of micromechanics-based material model into FEA solvers provides a feasible way in dealing with the global behavior of woven composite structures.

A number of analytical models were presented by many investigators. All of these models studied the average performance of a periodic representative volume cell. Ishikawa and Chou (1982, 1983a,b) suggested a variety of models to handle the in-plane behavior of woven composites. These are the mosaic model, the fiber undulation model, and the bridging model. The basic assumption for these models is that the classical lamination theory is valid for every infinitesimal strip of a representative cell. Ishikawa and Chou (1983c) also expanded their models to deal with constituent materials with shear nonlinearity and initial failure. Their work basically considers one-dimensional strip of a representative cell. As a result, these models cannot represent the material behavior of woven composites under bi-directional loading. Naik and Shembekar (1992a, b), Shembekar and Naik (1992), proposed a series-parallel model and a parallel-series model for considering the two-dimensional undulation geometry of plain woven composites.

The developed two-dimensional models to date provide a reasonable interpolation of material property prediction. However, there is still lack of local stress and strain information and no nonlinear stress-strain update procedure is available. To perform structural analysis of composite materials using the finite element method a standard interface must be developed. Rahman and Pecknold (1992) developed a micro-model for laminated composites and an interface with a finite element code. A similar interface with the nonlinear finite element code ABAQUS is developed and presented here for woven composite materials.

2. Mathematical formulation

Any non-hybrid plain weave fabric composite laminate can be represented by double-periodic representative volume cells, as shown in Fig. 1. The representative volume cell typically consists of two sets of interlaced yarns, known as fill and warp threads, and resins. The fill and warp threads, might be pure fibers or mixture of fibers and resins, are assumed to be homogenous and transversely isotropic. The resin is homogenous and isotropic. Given a representative volume cell, the global average stresses and strains can be written as:

$$\{\bar{\sigma}\} = \frac{1}{V}\iiint \{\sigma\}\,\mathrm{d}v \quad \text{and} \quad \{\bar{\varepsilon}\} = \frac{1}{V}\iiint \{\varepsilon\}\,\mathrm{d}v \tag{1}$$

The question that must be answered is that, given the average global stresses (or strains), what are the global strains (or stresses). For this purpose. a micromechanics-based model is developed on the basis of assumptions related to local stresses and strains.

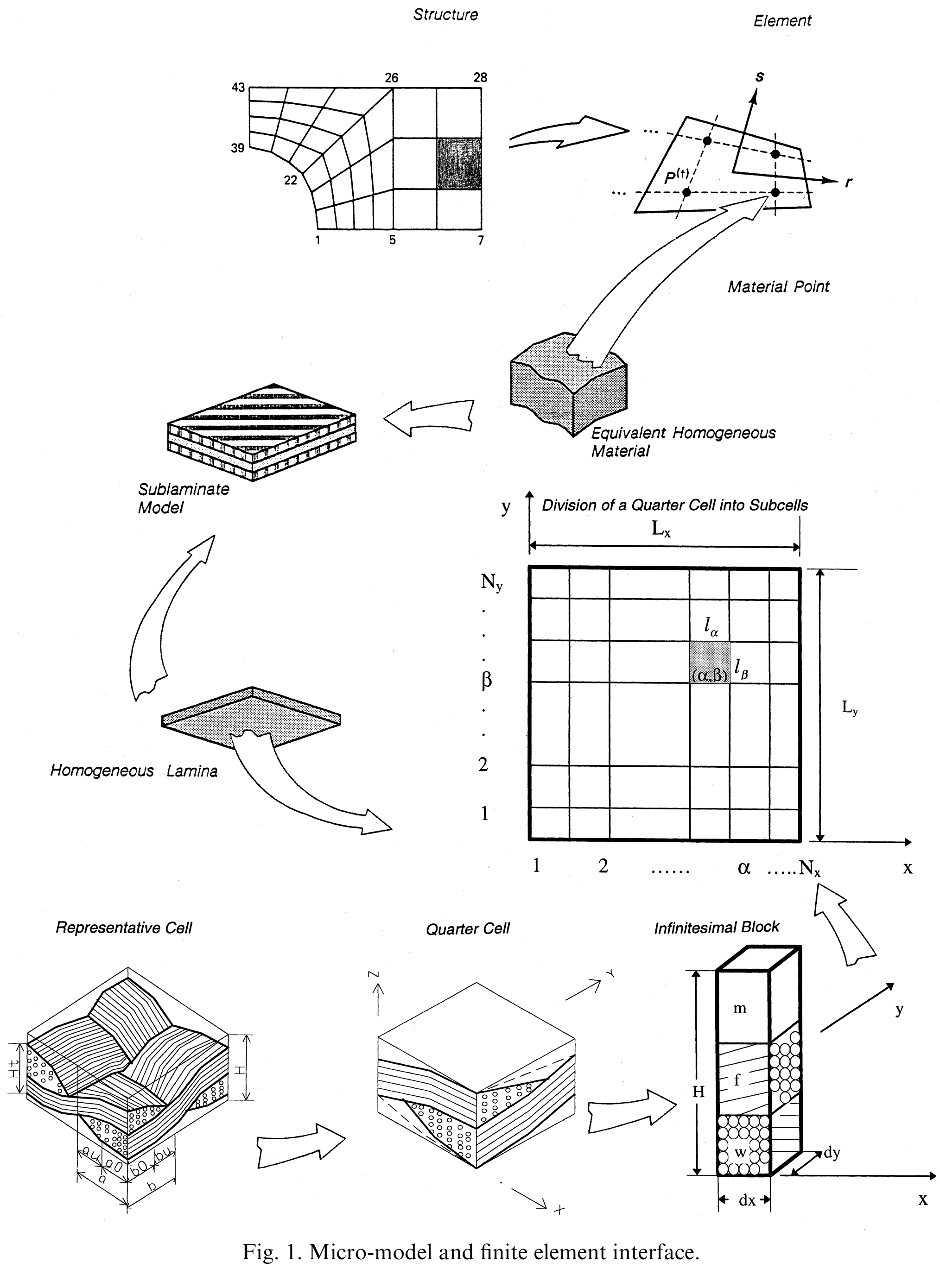

Fig. 1. Micro-model and finite element interface.

2.1. Effective stress–strain relations for an infinitesimal element

Consider a typical infinitesimal block of the representative cell with dx in length, dy in width and H in height, as shown in Fig. 1. This infinitesimal element generally consists of three different materials with $\bar{t}_m$, $\bar{t}_f$ and $\bar{t}_w$ volume fractions of matrix, fill and warp respectively. The material coordinate system of fills and warps may not coincide with the global coordinate system. Assuming that the in-plane relations for these three materials are in a parallel arrangement, the following equations are derived for incremental strains and stresses:

$$\mathrm{d}\{\varepsilon_i\} = \mathrm{d}\{\varepsilon_i\}_k \quad \text{(parallel)} \tag{2a}$$

$$\mathrm{d}\{\sigma_i\} = \sum_{k=m,f,w} \bar{t}_k\, \mathrm{d}\{\sigma_i\}_k \quad \text{(parallel)} \tag{2b}$$

where the subscript k denotes constituents with 'm' for matrix, 'f' for fill and 'w' for warp. $\{\varepsilon_i\}$ and $\{\sigma_i\}$ are the in-plane strains and stresses, respectively and the subscript i $= xx$, yy, xy. In the material coordinate system, the constitutive laws for matrix, fill and warp are all written as:

$$\begin{Bmatrix} \mathrm{d}\varepsilon_{11} \\ \mathrm{d}\varepsilon_{22} \\ \mathrm{d}\gamma_{12} \\ \mathrm{d}\varepsilon_{33} \\ \mathrm{d}\gamma_{13} \\ \mathrm{d}\gamma_{23} \end{Bmatrix}_k = \begin{bmatrix} S^t_{11} & S^t_{12} & 0 & S^t_{13} & 0 & 0 \\ S^t_{12} & S^t_{22} & 0 & S^t_{23} & 0 & 0 \\ 0 & 0 & S^t_{66} & 0 & 0 & 0 \\ S^t_{13} & S^t_{23} & 0S^t_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S^t_{55} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & S^t_{44} \end{bmatrix}_k \begin{Bmatrix} \mathrm{d}\sigma_{11} \\ \mathrm{d}\sigma_{22} \\ \mathrm{d}\sigma_{12} \\ \mathrm{d}\sigma_{33} \\ \mathrm{d}\sigma_{13} \\ \mathrm{d}\sigma_{23} \end{Bmatrix}_k \tag{3}$$

where the compliance components with superscript 't' represent the tangential ones. For symbolic derivation thereafter eqn (3) can be simply written as:

$$\begin{Bmatrix} \mathrm{d}\varepsilon^*_i \\ \text{—} \\ \mathrm{d}\varepsilon^*_o \end{Bmatrix}_k = \begin{bmatrix} S_i & | & S_{io} \\ \text{—} & \text{—} & \text{—} \\ S_{oi} & | & S_o \end{bmatrix}^t_k \begin{Bmatrix} \mathrm{d}\sigma^*_t \\ \text{—} \\ \mathrm{d}\sigma^*_o \end{Bmatrix}_k \tag{4}$$

where the subscript i $= 11, 22, 12$ and o $= 33, 13, 23$, respectively. In the global coordinate system, the constitutive relations can be written as:

$$\begin{Bmatrix} \mathrm{d}\varepsilon_i \\ \text{—} \\ \mathrm{d}\varepsilon_o \end{Bmatrix}_k = \begin{bmatrix} \psi_i & | & \psi_{io} \\ \text{—} & \text{—} & \text{—} \\ \psi_{oi} & | & \psi_o \end{bmatrix}^t_k \begin{Bmatrix} \mathrm{d}\sigma_i \\ \text{—} \\ \mathrm{d}\sigma_o \end{Bmatrix}_k \tag{5}$$

$$\begin{bmatrix} \psi_i & | & \psi_{io} \\ \text{—} & \text{—} & \text{—} \\ \psi_{oi} & | & \psi_o \end{bmatrix}^t_k = [T_1]_k \begin{bmatrix} S_i & | & S_{io} \\ \text{—} & \text{—} & \text{—} \\ S_{oi} & | & S_o \end{bmatrix}^t_k [T_1]^T_k \tag{6}$$

The expression of the transformation matrix $[T_1]_k$ can be found in Appendix 1 of Whitcomb (1991). In application of finite element method for analysis of the global behavior of thin-walled structures made of woven fabric composites, shell elements are usually employed. The required constitutive laws are generally written as follows:

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} \tag{7}$$

and

$$\begin{Bmatrix} \gamma_{23} \\ \gamma_{13} \end{Bmatrix} = \begin{bmatrix} S_{44} & 0 \\ 0 & S_{55} \end{bmatrix} \begin{Bmatrix} \sigma_{23} \\ \sigma_{13} \end{Bmatrix} \tag{8}$$

In these equations the global normal stress in thickness direction is ignored. During the incremental-iterative solution scheme, as used in finite element analysis of nonlinear problems, a change in the nodal displacements takes place. The displacement increment causes an increment of strain $\Delta\{\bar{\varepsilon}_i\}$ at a material point. The material model is required to calculate the tangential stiffness matrix and the incremental stress $\Delta\{\bar{\sigma}_i\}$. In this investigation, stiffnesses, strains, and stresses are tracked at the material points within each element. This information is provided by the woven composite material model, which interfaces with the nonlinear finite element code ABAQUS through the user defined subroutine UMAT. The woven heterogeneous nature of the material is hidden from the main analysis code. Figure 1 shows a schematic of the micromechanical model and the interface with the finite element code. Due to the symmetry of the representative unit cell, only quarter cell is considered. This quarter cell represents the same mechanical properties as the whole cell. The quarter cell is further divided into subcells. Each subcell is represented by an infinitesimal block as shown in Fig. 1. The actual woven composite is replaced by an equivalent homogeneous material whose properties are determined by requiring that the actual material and the equivalent material behave in the same way when subjected to certain stresses or strains. The interface consists of stresses and strains transfer between the material model and the analysis code. The main analysis code only sees this equivalent homogeneous anisotropic material.

In this investigation the averaging procedure employed yields the tangential stiffness matrix and incremental stress under an increment of strains. The material nonlinearity sought here is nonlinear shear induced by resins. As suggested by Hahn and Tsai (1973), the total nonlinear shear strain-stress relations for the matrix and fiber tows are presented in the following form:

$$(\gamma_{12})_k = (S_{66})_k(\sigma_{12})_k + (S_{6666})_k(\sigma_{12})_k^3 \tag{9a}$$

$$(\gamma_{13})_k = (S_{55})_k(\sigma_{13})_k + (S_{5555})_k(\sigma_{13})_k^3 \tag{9b}$$

In the above equations it is reasonable to assume that S_{6666} equals S_{5555} (Ishikawa and Chou, 1983c). It is also assumed that shear nonlinearity of the resin can adopt the same form as above. Neglecting normal stress in thickness direction and the coupling among in-plane normal components and transverse shear components, the in-plane stress–strain relations for off-axis fill tows, warp tows and matrix can be written as:

(1) Off-axis fill

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix}_{\mathrm{f}} = \begin{bmatrix} \psi_{11} & \psi_{12} & 0 \\ \psi_{12} & \psi_{22} & 0 \\ 0 & 0 & \psi_{66} \end{bmatrix}_{\mathrm{f}} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}_{f} + \begin{bmatrix} \psi_{11\mathrm{n}} & 0 \\ 0 & 0 \\ 0 & \psi_{66\mathrm{n}} \end{bmatrix}_{f} \begin{Bmatrix} (\sigma_x)^3 \\ (\sigma_{xy})^3 \end{Bmatrix}_{\mathrm{f}} \tag{10}$$

where

$$(\psi_{11})_{\mathrm{f}} = c^4(S_{11})_{\mathrm{f}} + 2c^2s^2(S_{13})_{\mathrm{f}} + s^4(S_{33})_{\mathrm{f}} + c^2s^2(S_{55})_{\mathrm{f}}$$
$$(\psi_{12})_{\mathrm{f}} = c^2(S_{12})_{\mathrm{f}} + s^2(S_{23})_{\mathrm{f}}$$
$$(\psi_{22})_{\mathrm{f}} = (S_{22})_{\mathrm{f}}$$
$$(\psi_{66})_{\mathrm{f}} = c^2(S_{66})_{\mathrm{f}} + s^2(S_{44})_{\mathrm{f}}$$
$$(\psi_{11\mathrm{n}})_{\mathrm{f}} = c^4s^4(S_{5555})_{\mathrm{f}}$$
$$(\psi_{66\mathrm{n}})_{\mathrm{f}} = c^4(S_{6666})_{\mathrm{f}} \quad \text{with } c = \cos(\theta_{\mathrm{f}}) \quad \text{and } s = \sin(\theta_{\mathrm{f}})$$

(2) Off-axis warp

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix}_{\mathrm{w}} = \begin{bmatrix} \psi_{11} & \psi_{12} & 0 \\ \psi_{12} & \psi_{22} & 0 \\ 0 & 0 & \psi_{66} \end{bmatrix}_{\mathrm{w}} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}_{\mathrm{w}} + \begin{bmatrix} 0 & 0 \\ \psi_{22\mathrm{n}} & 0 \\ 0 & \psi_{66\mathrm{n}} \end{bmatrix}_{\mathrm{w}} \begin{Bmatrix} (\sigma_y)^3 \\ (\sigma_{xy})^3 \end{Bmatrix}_{\mathrm{w}} \tag{11}$$

where

$$(\psi_{22})_{\mathrm{w}} = c^4(S_{11})_{\mathrm{w}} + 2c^2s^2(S_{13})_{\mathrm{w}} + s^4(S_{33})_{\mathrm{w}} + c^2s^2(S_{55})_{\mathrm{w}}$$
$$(\psi_{12})_{\mathrm{w}} = c^2(S_{12})_{\mathrm{w}} + s^2(S_{23})_{\mathrm{w}}$$
$$(\psi_{11})_{\mathrm{w}} = (S_{22})_{\mathrm{w}}$$
$$(\psi_{66})_{\mathrm{w}} = c^2(S_{66})_{\mathrm{w}} + s^2(S_{44})_{\mathrm{w}}$$
$$(\psi_{22\mathrm{n}})_{\mathrm{w}} = c^4s^4(S_{5555})_{\mathrm{w}}$$
$$(\psi_{66\mathrm{n}})_{\mathrm{w}} = c^4(S_{6666})_{w} \quad \text{with } c = \cos(\theta_{\mathrm{w}}) \quad \text{and } s = \sin(\theta_{\mathrm{w}})$$

(3) Matrix

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix}_{\mathrm{m}} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix}_{\mathrm{m}} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}_{m} + \begin{bmatrix} 0 \\ 0 \\ S_{6666} \end{bmatrix}_{m} (\sigma_{xy})^3_{\mathrm{m}} \tag{12}$$

Equation (5) can be written for the in-plane stresses in the following matrix form as:

$$\mathrm{d}\{\sigma_{\mathrm{i}}\}_k = [\psi^{\mathrm{t}}_{\mathrm{i}}]^{-1}_k \mathrm{d}\{\varepsilon_{\mathrm{i}}\}_k \tag{13}$$

Incorporating eqns (2) and (13), the following equation can be derived

$$\mathrm{d}\{\sigma_{\mathrm{i}}\} = [C_{\mathrm{i}}]\mathrm{d}\{\varepsilon_{\mathrm{i}}\} \tag{14}$$

where

$$[C_{\mathrm{i}}] = \sum_{k} \bar{t}_k [\psi_{\mathrm{i}}^{\mathrm{t}}]_k^{-1} \tag{15}$$

Finally, the effective stress–strain relations for the infinitesimal element are obtained by rearranging eqn (14) as follows:

$$\{\mathrm{d}\varepsilon_{\mathrm{i}}\} = [\Psi_{\mathrm{i}}]^{\mathrm{t}}\{\mathrm{d}\sigma_{\mathrm{i}}\} \tag{16}$$

where

$$[\Psi_{\mathrm{i}}^{\mathrm{t}}] = [C_{\mathrm{i}}]^{-1} \tag{17}$$

2.2. Average stress–strain relations for a subcell

The whole representative volume cell is divided into many subcells. It is assumed that stresses are uniformly distributed within each subcell. The incremental average stress–strain relations for a subcell can be obtained by performing integration of eqn (16), i.e.:

$$\{\mathrm{d}\varepsilon_{\mathrm{i}}^{(\alpha,\beta)}\} = [\bar{S}_{\mathrm{i}}]^{\mathrm{t}}\{\mathrm{d}\sigma_{\mathrm{i}}^{(\alpha,\beta)}\} \tag{18}$$

with

$$[\bar{S}_{\mathrm{i}}]^{\mathrm{t}} = \frac{1}{l_\alpha l_\beta}\int\!\!\int [\Psi_{\mathrm{i}}]^{\mathrm{t}}\,\mathrm{d}x\,\mathrm{d}y \tag{19}$$

From eqns (10)–(12) the incremental form of in-plane stress–strain relations of constituents may be written as:

$$\mathrm{d}\begin{Bmatrix}(\sigma_x)_k\\(\sigma_y)_k\\(\sigma_{xy})_k\end{Bmatrix} = \begin{bmatrix}(C_{11})_k & (C_{12})_k & 0\\(C_{12})_k & (C_{22})_k & 0\\0 & 0 & (C_{66})_k\end{bmatrix}^{\mathrm{t}}\mathrm{d}\begin{Bmatrix}(\varepsilon_x)_k\\(\varepsilon_y)_k\\(\varepsilon_{xy})_k\end{Bmatrix} \tag{20}$$

Once the incremental forms of the stress–strain relations of the constituents are obtained from eqn (20), the in-plane relations for a subcell can be derived from eqn (18) and are denoted by:

$$\mathrm{d}\begin{Bmatrix}\sigma_x^{(\alpha,\beta)}\\\sigma_y^{(\alpha,\beta)}\\\sigma_{xy}^{(\alpha,\beta)}\end{Bmatrix} = \begin{bmatrix}C_{11}^{(\alpha,\beta)} & C_{12}^{(\alpha,\beta)} & 0\\C_{12}^{(\alpha,\beta)} & C_{22}^{(\alpha,\beta)} & 0\\0 & 0 & C_{66}^{(\alpha,\beta)}\end{bmatrix}^{\mathrm{t}}\mathrm{d}\begin{Bmatrix}\varepsilon_x^{(\alpha,\beta)}\\\varepsilon_y^{(\alpha,\beta)}\\\varepsilon_{xy}^{(\alpha,\beta)}\end{Bmatrix} \tag{21}$$

In this procedure it is assumed that the average in-plane strains and stresses among subcells have the following relationships:

$$\mathrm{d}\sigma_x^{(\alpha,\beta)} = \mathrm{d}\sigma_x^{(\alpha,\beta^*)} \quad (\alpha = 1,\ldots,N_x, \beta = 1,\ldots,N_y-1, \beta^* = \beta+1) \tag{22a}$$

$$\mathrm{d}\sigma_y^{(\alpha,\beta)} = \mathrm{d}\sigma_y^{(\alpha^*,\beta)} \quad (\alpha = 1,\ldots,N_x-1, \beta = 1,\ldots,N_y, \alpha^* = \alpha+1) \tag{22b}$$

$$\mathrm{d}\sigma_{xy}^{(\alpha,\beta)} = \mathrm{d}\sigma_{xy}^{(\alpha,\beta^*)} \quad (\alpha = 1,\ldots,N_x, \beta = l,\ldots,N_y - 1, \beta^* = \beta + 1) \tag{22c}$$

$$\mathrm{d}\sigma_{xy}^{(\alpha,N_y)} = \mathrm{d}\sigma_{xy}^{(\alpha^*,N_y)} \quad (\alpha = 1,\ldots,N_x - 1, \alpha^* = \alpha + 1) \tag{22d}$$

$$\sum_{\alpha=1}^{N_x} \frac{l_x^{\alpha,\beta)}}{L_x} \mathrm{d}\varepsilon_x^{(\alpha,\beta)} = \mathrm{d}\bar{\varepsilon}_x \quad (\beta = 1,\ldots,N_y) \tag{22e}$$

$$\sum_{\beta=1}^{N_x} \frac{l_y^{(\alpha,\beta}}{L_y} \mathrm{d}\varepsilon_y^{(\alpha,\beta)} = \mathrm{d}\bar{\varepsilon}_y \quad (\alpha = 1,\ldots,N_x) \tag{22f}$$

$$\sum_{\beta=1}^{N_y} \sum_{\alpha=1}^{N_x} \frac{l_x^{(\alpha,\beta)}}{L_x} \frac{l_y^{(\alpha,\beta)}}{L_y} \mathrm{d}\varepsilon_{xy}^{(\alpha,\beta)} = \mathrm{d}\bar{\varepsilon}_{xy} \tag{22g}$$

where the quantities with 'bar' denote the incremental average strain stress components of the whole cell. The incremental average stresses induced by increment average strains are expressed as:

$$\mathrm{d}\{\bar{\sigma}_\mathrm{i}\} = \sum_{\alpha=1}^{N_x} \sum_{\beta=1}^{N_y} \frac{l_x^{(\alpha,\beta)} l_y^{(\alpha,\beta)}}{L_x L_y} \mathrm{d}\{\sigma_\mathrm{i}^{(\alpha,\beta)}\} \tag{23}$$

Equation (22) together with eqn (21) provides sufficient information to distribute the incremental average strains to each subcell. Once the average strain in each subcell is known, the incremental average stresses of the cell can be obtained by eqns (21) and (23). Simultaneously, one can obtain the tangential stiffness matrix. In the next section the calculation procedure will be provided. Now eqns (21) and (22) form a simultaneous linear system of equations, with incremental strains of each subcell as unknown and incremental average strains of the cell as known, presented by the following equation:

$$[B]\mathrm{d}\{\varepsilon_\mathrm{i}^{(\alpha,\beta)}\} = [K]\mathrm{d}\{\bar{\varepsilon}_i\} \tag{24}$$

with

$$\{\mathrm{d}\{\varepsilon_\mathrm{i}^{(\alpha,\beta)}\}\}^\mathrm{T} = \{\mathrm{d}\{\varepsilon_\mathrm{i}^{(1,1)}\}^\mathrm{T} \quad \mathrm{d}\{\varepsilon_\mathrm{i}^{(2,1)}\}^\mathrm{T}) \quad \mathrm{d}\{\varepsilon_\mathrm{i}^{(3,1)}\}^\mathrm{T} \quad \ldots \quad \mathrm{d}\{\varepsilon_\mathrm{i}^{(N_x N_y)}\}^\mathrm{T}] \tag{25}$$

From eqn (24), the incremental strains of each subcell can be solved for by the following:

$$\mathrm{d}\{\varepsilon_\mathrm{i}^{(\alpha,\beta)}\} = = [B]^{-1}[K]\mathrm{d}\{\bar{\varepsilon}_\mathrm{i}\} \tag{26}$$

The partitioned form of eqn (26) is

$$\mathrm{d}\{\varepsilon_\mathrm{i}^{(\alpha,\beta)}\} = [A^{\alpha,\beta)}]\mathrm{d}\{\bar{\varepsilon}_\mathrm{i}\} \quad (\alpha = 1,\ldots,N_x \quad \text{and } \beta = 1,\ldots,N_y) \tag{27}$$

By combining eqns (21), (23) and (27), the incremental average stresses can be obtained:

$$\mathrm{d}\{\bar{\sigma}_\mathrm{i}\} = \sum_{\alpha=1}^{N_x} \sum_{\beta=1}^{N_y} \frac{l_x^{(\alpha,\beta)} l_y^{(\alpha,\beta)}}{L_x L_y} [C^{(\alpha,\beta)}]^\mathrm{t} [A^{(\alpha,\beta)}]\mathrm{d}\{\bar{\varepsilon}_\mathrm{i}\} \tag{28}$$

or

$$d\{\bar{\sigma}_i\} = [\bar{C}]^t d\{\bar{\varepsilon}_i\} \tag{29}$$

with

$$[\bar{C}]^t = \sum_{\alpha=1}^{N_x} \sum_{\beta=1}^{N_y} \frac{l_x^{(\alpha,\beta)} l_y^{(\alpha,\beta)}}{L_x L_y} [C^{(\alpha,\beta)}]^t [A^{(\alpha,\beta)}] \tag{30}$$

Equation (30) provides the total tangential stiffness matrix for the average in-plane stress–strain relations. The incremental stresses calculated from eqn (28) accumulate error when the average incremental strains are not small enough. To overcome this problem one needs to update the strains in each subcell and then find the stresses of the constituents. The corresponding average stresses of the cell are directly determined by the following:

$$\{\bar{\sigma}_i\} = \sum_{\alpha=1}^{N_x} \sum_{\beta=1}^{N_y} \frac{l_x^{(\alpha,\beta)} l_y^{(\alpha,\beta)}}{L_x L_y} \{\sigma_i^{(\alpha,\beta)}\} \tag{31}$$

3. Numerical results

The constitutive equations are implemented in a user defined material subroutine UMAT in ABAQUS. As a demonstration example, numerical analysis of the stress–strain relations is performed for Glass/Polyimide plain weave composites. Basic material properties of constituents of the composite are listed in Table 1 (Ishikawa and Chou, 1983c). The geometry considered in the calculation is:

$$a_0 = b_0 = 0.0\ (mm), \quad a_u = b_u = a = b = 0.4\ (mm), \quad H = H_t = 0.244\ (mm)$$

The algorithm to calculate the constituent stresses of each subcell $\{\sigma_i^{(\alpha,\beta)}\}$ in eqn (31) can be summarized by the following:

- For the $n+1^{st}$ increment of global average strains $d\{\bar{\varepsilon}_i\}^{n+1}$, the corresponding incremental strains of each subcell are determined by eqn (27);
- The total strains of each subcell are updated by the following equation:

$$\{\varepsilon_i^{(\alpha,\beta)}\}^{n+1} = \{\varepsilon_i^{(\alpha,\beta)}\}^n + d\{\varepsilon_i^{(\alpha,\beta)}\}^n \tag{32}$$

- The total stresses for each constituent corresponding to the total average strains of each subcell

Table 1
Properties of Constituents (Gpa for moduli and Gpa^{-3} for S_{6666}) (Ishikawa and Chou, 1983c)

Glass/polyimide						Polyimide		
E_L	E_T	G_{LT}	υ_{LT}	v_{TT}	S_{6666}	E_m	v_m	S_{6666}
41.2	15.7	5.59	0.30	0.48	37.0	4.31	0.36	9.88

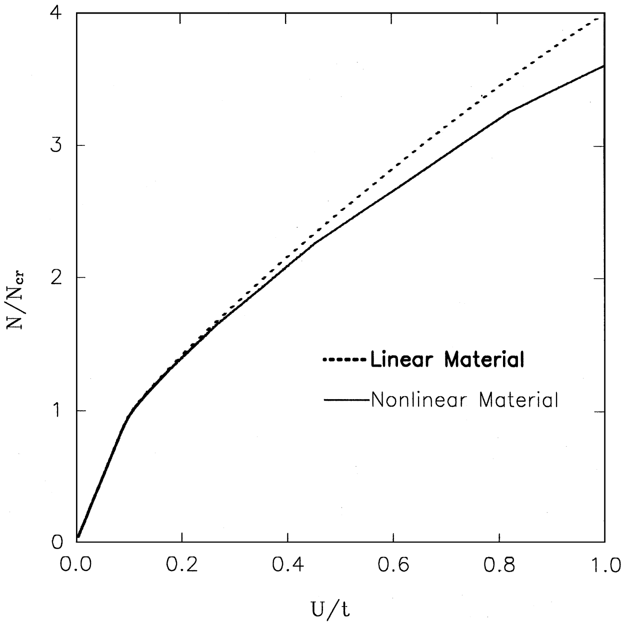

Fig. 2. Normalized post buckling load vs normalised end shortening for $(0)_{8s}$ plates.

are determined by solving eqns (10), (11) and (12). A nonlinear root-seeking scheme is applied to update the stresses of the constituents. Particularly, Newton's method is used here.

- The total stresses of each subcell are then obtained from the following equation:

$$\{\sigma_i^{(\alpha,\beta)}\}^{n+1} = t_m\{\sigma_i^{(\alpha,\beta)}\}_m^{n+1} + t_f\{\sigma_i^{(\alpha,\beta)}\}_f^{n+1} + t_w\{\sigma_i^{(\alpha,\beta)}\}_w^{n+1} \tag{33}$$

where the subscripts 'm', 'f', and 'w' denote matrix, fill and warp, respectively.

In order to demonstrate the applicability of the developed methodology for nonlinear finite element, a nonlinear post buckling analysis of woven composite plates is conducted. In addition, the numerical simulation is performed to determine the effect of material nonlinear behavior of the post buckling of woven composite plates. Figures 2–7 depict the post buckling behavior of several plates with different orientations of the woven composites and plate thickness (N_{cr} is the bifurcation load for the plates). Figure 8 shows the effect of plate thickness (number of plies) on the normalized post buckling load. The post buckling load considered for demonstration of material nonlinear effect is taken as $U/t = 0.5$ (N_{linear} is the value of axial load for the case of plates with linear material behavior). Only two cases are considered here, 16 and 32 plies. We can observe that as the plate thickness increases the effect of material nonlinearity becomes more pronounced.

4. Conclusion

A stress update procedure is developed. A stress/strain averaging procedure is presented for predicting the nonlinear behavior of woven composites. The presented methodology can be directly applied to nonlinear finite element codes for structural analysis as demonstrated in the investigation.

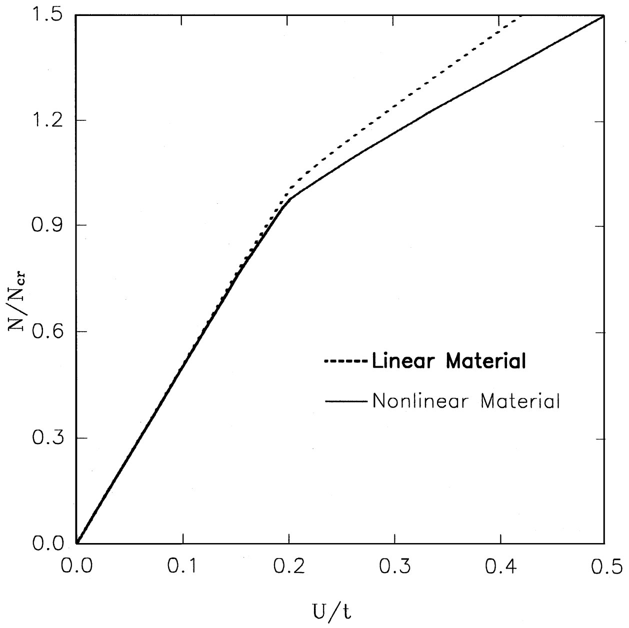

Fig. 3. Normalized post buckling load vs normalized end shortening for $(0)_{16s}$ plates.

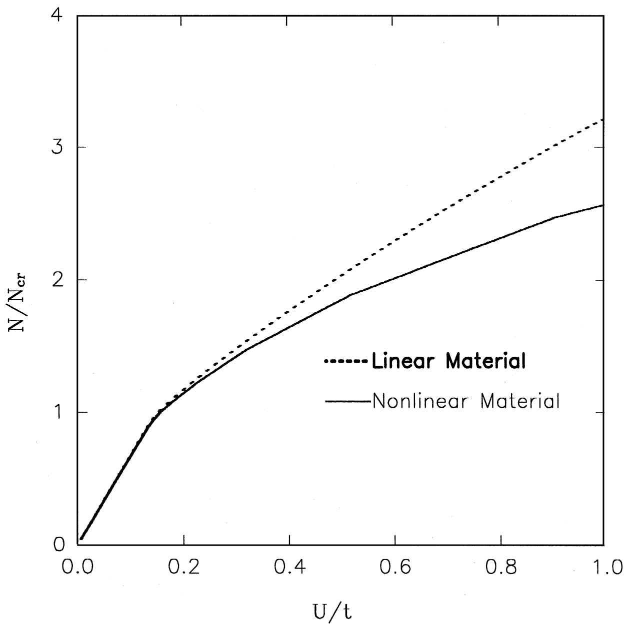

Fig. 4. Normalized post buckling load vs normalized end shortening for $(\pm 45)_{4s}$ plates.

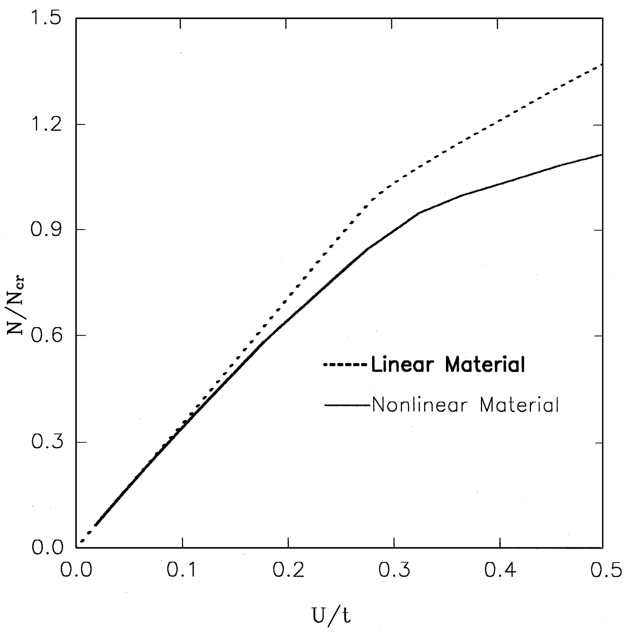

Fig. 5. Normalized post buckling load vs normalized end shortening for $(\pm 45)_{8s}$ plates.

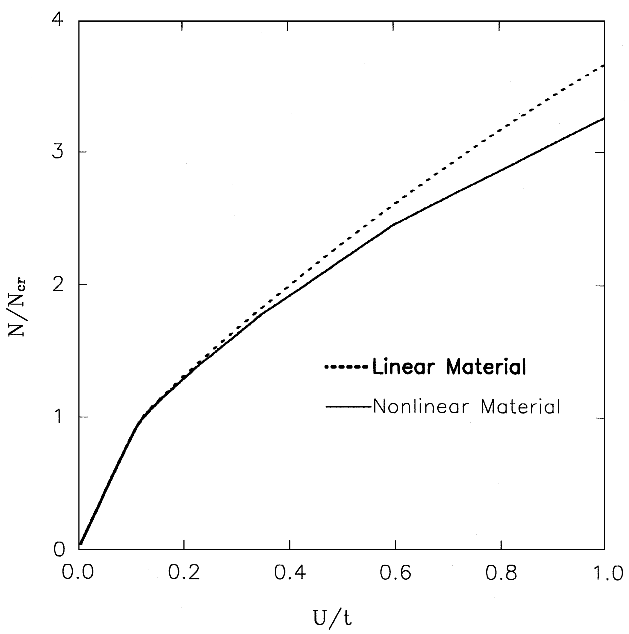

Fig. 6. Normalized post buckling load vs normalized end shortening for $(0/\pm 45/90)_{2s}$ plates.

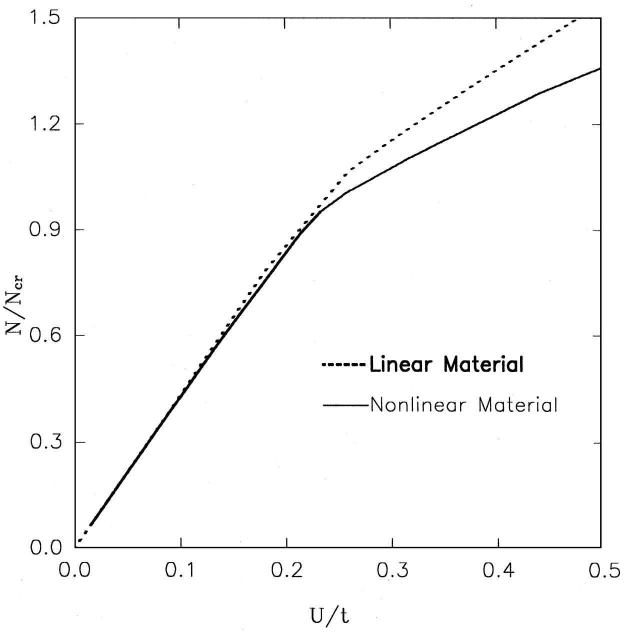

Fig. 7. Normalized post buckling load vs normalized end shortening for $(0/\pm45/90)_{4s}$ plates.

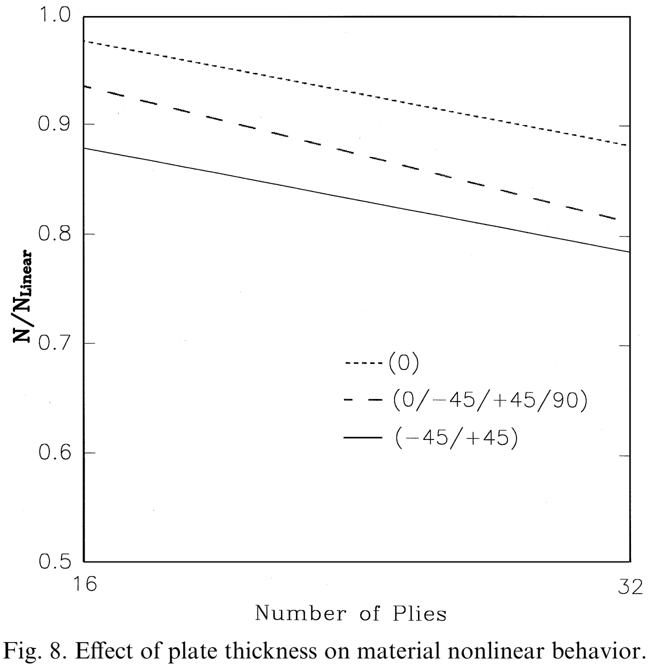

Fig. 8. Effect of plate thickness on material nonlinear behavior.

The shear nonlinearity, which exists in the majority of polymer composites, is also included in the numerical simulation. It is found that slight nonlinearity exists between the uniaxial stress and strain relation for fabric woven composites. On the other hand, shear nonlinearity is much more pronounced. The developed formulation is not yet experimentally validated. Validation of such methodology is costly and time consuming since tests must be conducted on the constituents' level. The validation study is planned for the future once proper experimental data is found.

The ultimate use of the developed methodology is for failure analysis. This can be accomplished by defining Micro-Failure Criteria (MFC) for determination of various failure modes since stresses and strains in each subcell and each constituent is available at each load increment.

Acknowledgements

The research was partially sponsored by the Office of Naval Research, Ship Structure Division, with Dr Y. D. S. Rajapakse as scientific officer. The valuable discussions and suggestions of Professor G. Simitses, principle investigator of the ONR Grant are gratefully acknowledged. Computing support was provided by the Ohio Super Computer Center. Their support is gratefully acknowledged.

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